

## Coombs' Theory of Unfolding

Coombs' theory is similar to Thurstone's in that it also was proposed in order to support a scaling procedure (in this case the "unfolding" procedure, from which the theory derived its name). Hence, it presumes the existence of quantitative variables of a psychological kind. It differs from Thurstone's theory in two important respects. In the first place it concerns a special category of pair comparison judgments, namely, preferences. Second, it lacks the probabilistic mechanism inherent to Thurstone's theory. Consider the situation where a person is making pair comparison preference judgments on a set of attitude statements. The subject is not judging which statement within each pair is the more favorable; he is indicating his preference (i.e., which statement within each pair he agrees with most). Assume that the statements lie along a single relevant quantitative dimension (e.g., the pro-anti dimension). Let each person have a point of maximum preference on this dimension. Then according to Coombs' unfolding theory a person will prefer statement  $x$  to statement  $y$ , if and only if he perceives his point of maximum preference to be nearer to  $x$  than to  $y$  along this dimension. Thus each pair comparison preference judgment provides information about the relative distances between a subject's point of maximum preference and the stimuli used. Because the theory postulates that all subjects agree about the location of the stimuli on this dimension, even though they might have different points of maximum preference, pair comparison preference judgments from different subjects may be pooled to provide enough distance information to infer something of the quantitative structure of this dimension. Ideally, both the location of the stimuli and the subjects' points of maximum preference may be inferred. Something of the flavor of this procedure is given in this chapter, although the primary concern is with the logic of the theory: its assumptions, testability, and use as a basis for psychological measurement.

The theory and its associated method were first proposed by Coombs in 1950. He subsequently integrated it into a much broader theory of judgmental behavior in his book, *A Theory of Data* (1964). The procedure was moderately well received and included amongst the prescribed methods of psychological measurement (e.g., Green, 1954; & Torgerson, 1958). It has the advantage of fitting very neatly into the representational measurement theory framework. A description of the procedure is included in a number of recent scaling text books (e.g., Dawes, 1972 & van der Ven, 1980). At first sight it is somewhat surprising then that in the last 30 or more years the number of its applications is very few. Its inclusion in the books mentioned is based more on promise than performance. The reason is not hard to find and it provides an interesting commentary on the attitude of psychologists towards measurement. The successful application of Coombs' theory demands that the stimuli used actually be unidimensional. That is, that there be only a single variable relevant to the subjects' judgments. Thus, its application requires careful stimulus control in contexts where such control is very difficult to attain. This in turn requires intensive research into relevant stimulus attributes. In general psychologists have shied away from that daunting task and found refuge in quantitative methods that, because they assume more, demand less foundational research as the basis for their application. Methods that always yield a scaling solution, like the method of summated ratings, are almost universally preferred to methods which, like Coombs', do not produce a scaling solution when they are falsified by the data. Surprisingly, vulnerability to falsification is commonly deemed by psychologists to be a fault rather than a virtue.

## AN OUTLINE OF THE THEORY

Coombs' theory may be summed-up in the following five propositions.

- (1) There is a single stimulus variable relevant to the preference judgments.
- (2) That variable is quantitative.
- (3) Each subject has a single point of maximum preference on that variable.
- (4) A person's degree of preference for any stimulus,  $x$ , decreases as the distance between  $x$  and the person's point of maximum preference increases.
- (5) All subjects agree about the location of the stimuli on the relevant dimension.

Coombs calls a person's point of maximum preference his or her *ideal point* and he calls a person's preference ordering of the stimuli an *I scale* (the "I" indicating that it is the order of an individual subject). The use of the term "scale" to denote an ordering is too grandiose. Hence, the more accurate descriptive phrase "individual preference order" is used here instead.

The kind of situation the theory covers is illustrated geometrically in Fig. 7.1. In that figure there are four stimuli,  $A$ ,  $B$ ,  $C$ , and  $D$  and their values on the relevant variable are represented as points on a line. Person  $i$ 's point of maximum preference on the variable is indicated by the arrow. Coombs calls such a geometric representation of both stimuli and individuals a *J scale* (the "J" indicating the diagram's joint accommodation of stimuli and individuals). Person  $i$ 's preference ordering of the four stimuli would be  $BACD$ , because that is the order of their proximity to  $i$ 's ideal point. On the other hand, another person,  $j$ , whose ideal point is to the right of the  $CD$  midpoint, would give  $DCBA$  as their order of preference. Coombs suggests that a person's preference order may be obtained by folding the line representing the variable at the person's ideal point so that one is left with a single line, as in Fig. 7.2. Then the person's preference order is simply the order of the stimuli down this line. Because one does not know the locations of the stimuli on the variable this is not, of course, a real possibility. The aim of Coombs' procedure is to discover something of these locations from the individual preference orderings that constitute the data. However, at the theoretical level, these preference orderings are "foldings" of the  $J$  scale. Coombs' procedure, he suggests, may be thought of metaphorically as a simultaneous "unfolding" of them. Hence, the name given to his procedure and the theory sustaining it.

Before considering the application of conjoint measurement theory to unfolding theory, the five propositions constituting the theory will be considered in more detail. Proposition 1 locates the type of stimulus domain to which the theory applies. It applies only to unidimensional stimuli, that is, to stimuli varying along a single dimension relevant to the preferences of the subjects involved. If the stimuli vary along more than one relevant dimension then the theory does not apply. Thus, Proposition 1 is not so much a part of the theory as a boundary condition necessary for its application. In multidimensional contexts a different, though related, theory might be contemplated and, indeed, Coombs (1964) has done just that.

Proposition 2 requires that the relevant variable on which the stimuli differ be a quantitative variable. As with 1, this may also be regarded as setting a boundary condition for the application of the remainder of the theory. Thus, an adequate test of Coombs' theory would require construction of stimuli satisfying these two conditions. Then, failure of the theory's predictions would imply its falsity. Many

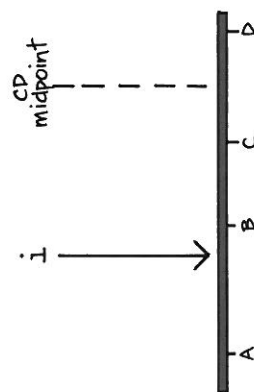


FIG. 7.1. Four stimuli differing along a single dimension relevant to person  $i$ 's preferences. According to Coombs' theory  $i$ 's order of preference will be:  $BACD$ . Another person, to the right of, say, the  $CD$  midpoint would give  $DCBA$  as their preference order.

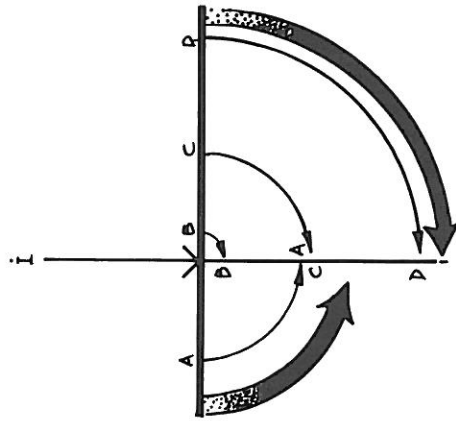


FIG. 7.2.  $i$ 's preference order may be obtained by folding the relevant dimension at  $i$ 's ideal point to form a single vertical line.  $i$ 's order of preference is the order of the stimuli down this line.

of its current applications within psychology, as in attitude measurements for example, beg Propositions 1 and 2.

Proposition 3 requires that each subject has a *single* point of maximum preference on the relevant variable. This is a simple and attractive hypothesis but by no means logically necessary. It is false, for example, if a subject has *no point* of maximum preference. Suppose a subject is choosing between the members of a set of paintings. Further suppose that this person is a complete philistine for whom the only relevant variable is monetary value. Then he may not have a single point of maximum preference simply because there is no greatest monetary value. Another person may not have a point of maximum preference on this variable because he completely disregards this variable in making his choices. However in that case it is not a relevant variable for him.

Proposition 3 is also false if any subject has more than a single point of maximum preference. This is a real possibility. Proposition 3 denies the complexity of human beings; the fact that we are often torn between competing interests. For example, in choosing something as simple as a cup of tea a person might have two quite distinct points of maximum preference. Imagine a set of cups of tea that differ only with respect to quantity of sugar. The same person may maximally prefer tea with no sugar (because of his dietary interests), and tea with two teaspoons of sugar (because of taste). Could such a person make a choice? A prediction would depend on our theory and it's not clear what Coombs' theory would predict. Clearly, this is an important issue and one that requires a deeper analysis of just how our preferences are determined by our interests, and recognition of the fact that interests within the same person often conflict.

Proposition 4 assumes the truth of 3 and relates degree of preference to distance from the subject's point of maximum preference. It has two important facets. First, the degree of preference decreases as distance from the point of maximum preference increases. Second, because it makes no mention of direction it follows

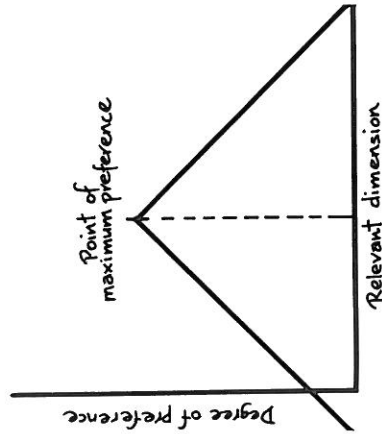


FIG. 7.3. Degree of preference is monotonic decreasing and symmetric about the point of maximum preference (ideal point) according to Coombs' theory.

that the rate of decrease must be the same on either side of the point of maximum preference (as shown in Fig. 7.3). Of course, the decrease need not be linear but it must be symmetric.

Proposition 5 is necessary because the location of the stimuli on the relevant dimension is inferred from the judgments of a number of different subjects. Hence, they cannot disagree about where the stimuli are located, or else the stimulus locations inferred will not necessarily be representative of the subjects. For example, if one was attempting to locate political candidates on the left-right political dimension, and the subjects disagreed about the location of different candidates, then there would be no common scale that represented the views of all subjects. Obviously, this proposition is the most tenuous of all when the stimulus variables are of a psychological nature.

## THE APPLICATION OF CONJOINT MEASUREMENT

Coombs' theory may be expressed as follows. Let  $S$  be a set of stimuli varying with respect to a single relevant dimension which is quantitative. Then for any person,  $a$ ,

$$P(a, s, t) \text{ if and only if } |a - s| < |a - t|$$

(where  $P(a, s, t)$  is the ternary relation, person  $a$  prefers  $s$  to  $t$ , and the values of  $a$ ,  $s$ , and  $t$  on the dimension are symbolized by those letters). The form of the inequality on the right is similar to that mentioned in relation to the application of conjoint measurement to the postulate of intradimensional subtractivity in the last chapter. It may be thought that conjoint measurement applies to Coombs' theory in a similar way. However, there are some complications in this approach and a simpler line will be taken. This simpler alternative depends on the fact that given any body of data (i.e., preference orderings from a group of subjects), Coombs' theory implies an ordering on the interstimulus midpoints, and because

these midpoints are an additive function of the stimulus values, this ordering must satisfy the hierarchy of cancellation conditions.

Let  $S$  be a set of  $n$  stimuli differing with respect to a single relevant quantitative variable. Let  $A$  be a group of subjects and let each subject produce a preference ordering of the stimuli in  $S$ . For the moment the possibility of ties in these preference orderings will be ignored, and it will be assumed that for each subject,  $a$ , there is a preference ordering of the  $n$  stimuli,  $P_a = \{s, t, \dots\}$ , such that any stimulus,  $x$ , comes before any other stimulus,  $y$ , if and only if  $P(a, x, y)$ .

If there are  $n$  stimuli then  $n!$  different preference orders are possible. Coombs' theory, however, predicts that no more than  $\frac{1}{2}n(n-1) + 1$  of these will occur. The reason is as follows. First, consider a simple illustration. Figure 7.4 shows the location of four hypothetical stimuli,  $A$ ,  $B$ ,  $C$ , and  $D$  along the relevant dimension. It also shows the interstimulus midpoints (the bisectors have been drawn in). Because there are four stimuli there are  $\frac{1}{2}n(n-1) = \frac{1}{2} \cdot 4(4-1) = 6$  interstimulus midpoints and they partition the dimension into  $6 + 1 = 7$  intervals. Associated with each interval is a unique preference order because if the points of maximum preference from any two subjects fall within the same interval then the relative proximity of the stimuli to their points of maximum preference is the same. Hence, in this case there must be only seven different preference orders allowed by Coombs' theory. More generally, if there are  $n$  stimuli at various different points along a single dimension then there must be  $\frac{1}{2}n(n-1)$  interstimulus midpoints, and so  $\frac{1}{2}n(n-1) + 1$  intervals separated by the midpoints. Each interval has associated with it its own preference order. Hence, there are no more than  $\frac{1}{2}n(n-1) + 1$  allowed.

Not only does Coombs' theory predict that no more than  $\frac{1}{2}n(n-1) + 1$  of the  $n!$  possible preference orders will occur, it also predicts that those that do occur have a special structure. Note that in Fig. 7.4 as one moves from left to right each new preference order differs from the preceding one by just the inversion of a single pair of stimuli, and this pair is just those whose midpoint was last crossed. This pattern continues until all midpoints are crossed and the final preference order becomes the complete reversal of the first. So the  $\frac{1}{2}n(n-1) + 1$  permissible preference orders must have that kind of structure.

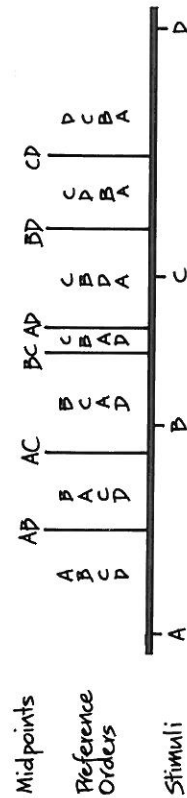


FIG. 7.4. The position of four hypothetical stimuli along the relevant dimension. Also shown are the six dashed lines bisecting the distances between the pairs of stimuli; the seven intervals so created; and the preference orders that, according to Coombs' theory, belong to any person whose point of maximum preference falls within any such interval.

More can be said about this structure. Notice that in Fig. 7.4 the only two preference orders that are complete reversals of each other are the first and last. Notice also that each of these gives the order of the stimuli along the dimension, either from right to left, or from left to right. So, according to Coombs, if any body of preference data contains such a pair of orders then they give the order of the stimuli along the dimension. In turn, the order of the stimuli along the dimension gives part of the order of the interstimulus midpoints along the dimension. For example, if, as in Fig. 7.4, the order of the stimuli along the dimension is  $A, B, C$ , and  $D$  then it follows that the order of the midpoints must include  $AB < AC < AD$  and  $BC < BD$  (where " $AB$ ", " $AC$ ",  $\dots$  etc. refer to midpoints and " $<$ " means "comes before"). This can be illustrated in Fig. 7.5, where there is a matrix whose rows and columns are the four stimuli listed in their order along the dimension and the cell entries (above the diagonal) refer to the midpoints. These cells must be ordered from left to right, and from top to bottom. Hence, they will satisfy single cancellation (or independence). Of course, this does not completely order all of the midpoints, the order between the  $BC$  and  $AD$  midpoints being unstated. But that order is settled by the fact that the preference order  $CBAD$  is present, rather than  $BCDA$ : the  $AD$  midpoint comes before the  $BC$  midpoint.

The preference orders satisfying the constraints illustrated in Fig. 7.5 are set out in Fig. 7.6. The are joined by arrows and beside each arrow is the stimulus pair reversed in moving from the preference order above to the one below. Note that when we get to  $BCAD$  there are two possibilities: either  $AD$  or  $BC$  may be reversed. The path we take depends on the data. If  $CBAD$  is given by some subjects then  $BC < AD$  and if  $BCDA$ , then  $AD < BC$ . Coombs' theory predicts that both cannot occur.

In general terms then, if  $x_1, x_2, \dots, x_n$  is the order of the  $n$  stimuli along the relevant dimension, then for any  $j, k = 1, 2, \dots, n$  ( $j \neq k$ ):

$$x_j < x_k \text{ if and only if } x_j x_k < x_k x_j$$

|   | A | B  | C  | D  |
|---|---|----|----|----|
| A | . | AB | AC | AD |
| B |   | .  | BC | BD |
| C |   |    | .  | CD |
| D |   |    |    | .  |

FIG. 7.5. The order imposed on the six interstimulus midpoints in Fig. 7.4 because the order of the stimuli along the relevant dimension is  $A, B, C$ , and  $D$ .



|   | A | B  | C  | D  | E  | F  |
|---|---|----|----|----|----|----|
| A |   | AB | AC | AD | AE | AF |
| B |   |    | BC | BD | BE | BF |
| C |   |    |    | CD | CE | CF |
| D |   |    |    |    | DE | DF |
| E |   |    |    |    |    | EF |
| F |   |    |    |    |    |    |

FIG. 7.8. If the order of the stimuli along the relevant dimension is  $A, B, C, D, E$ , and  $F$ , then the preference orders  $BCDEAF, DCBEAF$ , and  $DCEBFA$  jointly imply this order upon the interstimulus midpoints (the arrow goes from earlier to later values).

direction of the arrows in Fig. 7.8, this means that the double cancellation condition is violated and violation of that condition means that there is no way that  $A, B, C, D, E$ , and  $F$  can be placed along the relevant dimension (in that order), so that these three preference orders can be accommodated.

This can be easily seen algebraically. If the  $AE$  midpoint precedes the  $BD$  then  $\frac{1}{2}(A + E) < \frac{1}{2}(B + D)$  and that means,

$$A + E < B + D$$

$$\text{i.e. (i) } E - D < B - A.$$

Similarly,  $\frac{1}{2}(C + D) < \frac{1}{2}(A + F)$  implies that

$$\text{(ii) } C - A < F - D$$

and  $\frac{1}{2}(B + F) < \frac{1}{2}(C + E)$  implies that

$$\text{(iii) } F - E < C - B.$$

But given that the order long the dimension is  $A, B, C, D, E, F$  it follows that

$$C - A = B - A + C - B$$

$$\text{and } F - D = E - D + F - E,$$

so, as shown in Fig. 7.9, if (i) and (iii) are satisfied (ii) cannot be.

What now must be obvious is that if any of the hierarchy of cancellation conditions fails then Coombs' theory proves false. So conjoint measurement provides an ideal way to test the theory. As well, given that Coombs' theory is true and the requisite degree of stimulus control attained, it would prove the ideal way to quantify psychological variables lying behind preferences. This would involve the construction of standard sequences of stimuli based on equality of midpoints of nested pairs. In Fig. 7.9 for example, the  $BC$  pair is nested between the  $AD$  pair. If their midpoints coincided then  $D - C = B - A$ . A complete

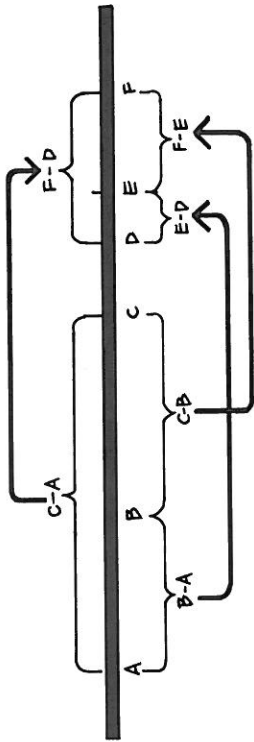


FIG. 7.9. If  $B - A$  exceeds  $E - D$  and  $C - B$  exceeds  $F - E$  then  $C - A$  must exceed  $F - D$ .

standard sequence for the six stimuli  $A, B, C, D, E$ , and  $F$  is shown in Fig. 7.10. Here the coincidence of the  $AF, BE$ , and  $CD$  midpoints implies that

$$E - D = C - B,$$

$$\text{and } F - D = C - A,$$

$$\text{whence } F - E = B - A.$$

Also, the coincidence of the  $AD$  and  $BC$  midpoints means that  $D - C = B - A$  and the coincidence of the  $CF$  and  $DE$  midpoints means that  $F - E = D - C$ . Finally, the coincidence of the  $BD$  and  $AE$  midpoints means that  $E - D = B - A$  and the coincidence of the  $CE$  and  $BF$  midpoints means that  $F - E = B - C$ . Thus,  $B - A = C - B = D - C = E - D = F - E$ . That is, we have a standard sequence.

In the case of Coombs' theory evidence for a standard sequence is obtained by getting fewer than the allowed number of individual preference orders. Note that in Fig. 7.10, only 10 of the 16 permitted orders is present. So the existence of a standard sequence puts further constraints on the occurrence of individual preference orders.

I am not suggesting that the construction of such sequences would be easy. Far from it. The important point is that their construction would provide compelling evidence that the variable involved is quantitative.

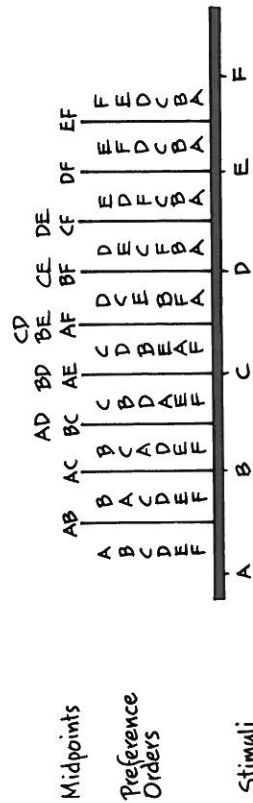


FIG. 7.10. A hypothetical standard sequence of six stimuli.

## AN EXAMPLE OF CONJOINT MEASUREMENT

In his book *A Theory of Data*, Coombs reports some data which will suffice to illustrate the conjoint measurement analysis of his theory. He instructed 62 students at the University of Michigan to make judgments on all pair comparisons on the letter grades  $A +$ ,  $A -$ ,  $B +$ ,  $B -$ ,  $C +$ , which were grades they might get in their course. For each pair comparison the students were asked which they thought they were more likely to get. 58 of the students gave transitive judgments, while four did not. The data from these four subjects consequently violates Coombs' theory, because it predicts that each subject's judgments will be transitive. However, each intransitivity could have resulted from just one error. In that case, there would be four errors out of 1302 pair comparison judgments and any theory can live with that.

For ease of presentation, let the grades  $A$  to  $C+$  be represented by the symbols  $A$  to  $G$  in the obvious order. If the order of the stimuli along the dimension is  $A, B, C, D, E, F, G$  then Coombs' theory predicts that each preference order will be consistent with that ordering, in the sense that it can be produced by "folding" that order at some point, and interleaving the stimuli in the two folded arms.

This sense of consistency is worth stating precisely. Let  $x_1, x_2, \dots, x_n$  be the order of the stimuli along the relevant dimension (i.e.,  $x_1 < x_2 < \dots < x_n$ ). Let the preference ordering for any subject,  $a$ , be  $P_a = x_i, \dots, x_j, \dots, x_k, \dots$ . Then Coombs' theory implies that for each preference order,  $P_a$ ,

- (i) for all  $j, k > i$ ,  $x_j$  comes before  $x_k$  in  $P_a$  if and only if  $j < k$ ; and
- (ii) for all  $j, k < i$ ,  $x_j$  comes before  $x_k$  in  $P_a$  if and only if  $j > k$ .

In general, if there are  $n$  stimuli, there will be  $2^{n-1}$  preference orders consistent with any given ordering of those stimuli along the dimension. In this case  $2^{n-1} = 2^6 = 64$  and the set of 64 preference orders consistent with the stimulus order,  $A, B, C, D, E, F$ , and  $G$  along the dimension is shown in Fig. 7.11. Those preference orders obtained by Coombs are shown in Fig. 7.12 (with their frequencies of occurrence in parentheses). All of the 58 subjects whose preference orders were transitive produced preference orders that were consistent with the stimulus order mentioned. This result suggests that all of the subjects agreed that was the order of the stimuli along the dimension.

It is also worth mentioning that each of the preference orders given by the four intransitive subjects could be made transitive, and converted to one of these 64 permitted orders by an alteration to only one of the pair comparison judgments made. It is in this sense that so far there are only 4 "errors" out of 1302 judgments.

Of course, Coombs' theory predicts that not only must the preference order produced come from those listed in Fig. 7.11, they must all fall on a single path through that diagram. Each path through this figure accommodates no more

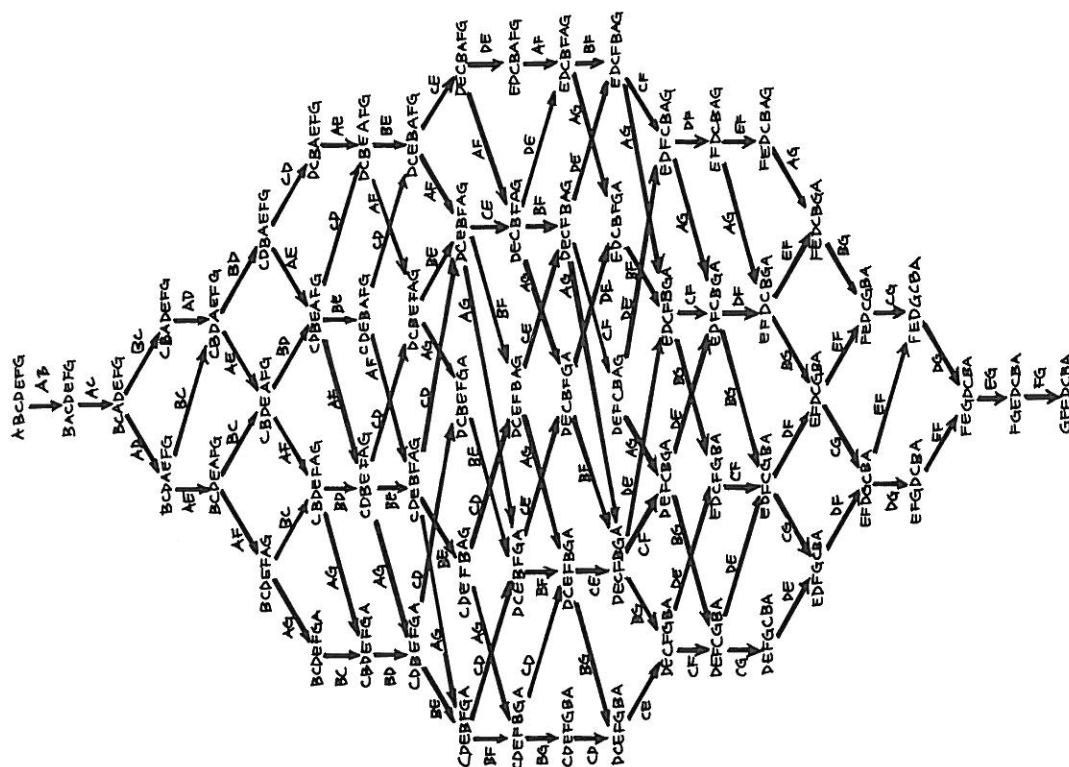


FIG. 7.11. The preference orders allowed by Coombs' theory form a single descending path through this graph, given that the order of the six stimuli along the relevant dimension is  $A, B, C, D, E, F$ , and  $G$ .



happens it implies the existence of additional tests of double cancellation. These additional tests are illustrated in Fig. 7.15. They include no violations of double cancellation.

Satisfaction of both single and double cancellation in the data shown in Fig. 7.13 raises the question of whether or not any further tests of higher order cancellation conditions must be performed. While the matrix,  $ABCD \times DEFG$ , within that figure is not a complete  $4 \times 4$  matrix (the order of the  $D, D$  cell relative to the others is not known), it is possible to test triple cancellation in  $3 \times 4$  or  $4 \times 3$  matrices, and there are two such matrices in Fig. 7.13. At this point in the testing procedure it would be helpful to know exactly which tests of triple cancellation are logically independent of double cancellation. It would then be known which tests of triple cancellation, if any, need to be performed in order to verify or falsify the claim that the data in Fig. 7.13 satisfies the hierarchy of cancellation conditions. However, as the answers to these questions are not known, a different approach must be used.

One approach that may be used is relatively simple and convenient. It involves making numerical estimates of the stimuli ( $A, B, C, D, E, F, G$ ) on some arbitrary scale that are consistent with the order on the midpoints given in Fig. 7.13. If such estimates can be made then the order in Fig. 7.13 must satisfy the hierarchy of cancellation conditions. There are a variety of techniques that produce such estimates. Coombs (1964, pp. 96-102) explains one (see also McClelland & Coombs, 1975) that may be conveniently done by hand. The values he obtained for this particular set of data are shown in Table 7.1.

The numerical order on the cells in Table 7.1 does not contradict the order shown in Fig. 7.13, though necessarily it is not exactly the same because some midpoints unordered in Fig. 7.13 are ordered in Table 7.1. However, this result is sufficient to show that the data in Fig. 7.13 must satisfy all the cancellation conditions that apply to it.

Simple as this procedure may be, it suffers two drawbacks. First, if a numerical solution consistent with the prescribed order on the midpoints does not exist, then it is not known which, or how many, independent tests of the cancellation conditions fail. Although some numerical estimation procedures do provide a solution of "best fit" in such circumstances (and some even provide a "measure" of how close this solution is to the prescribed order) it would be useful to know just which cancellation conditions fail.

Second, there is a temptation to regard these numerical estimates as genuine measurements. However, to treat them as measurements is premature, for they are only measurements if the variable involved is quantitative and Coombs' theory is true. In this particular instance 98% of the pair comparison preferences agreed with Coombs' theory. This may be taken as confirmation of the theory. However, the theory requires a more searching experimental analysis than this before being accepted as true, and until such an analysis is made any numerical estimates made must not be thought of as measurements. To do so is to succumb to wishful thinking.

to the later midpoint. The direction of the arrows is determined by their direction in Fig. 7.13. Note that in (a) and (f), the direction of the conclusion is not known because the order of those interstimulus midpoints was not determined by the data. Consequently, these two matrices do not permit a test of the theory. The other five, however, do and reference to Table 5.3 shows that they all satisfy that condition. Fig. 7.14 (b) and (c) are instances of pattern 9 in Table 5.3, (d) is an instance of pattern 1 and (e) and (g) are instances of pattern 13. These are all confirmatory patterns.

It is interesting to note a special feature of the matrix in Fig. 7.12, a feature which has implications for the testing of double cancellation. This feature is that the same seven stimuli define both the rows and the columns. Whenever this

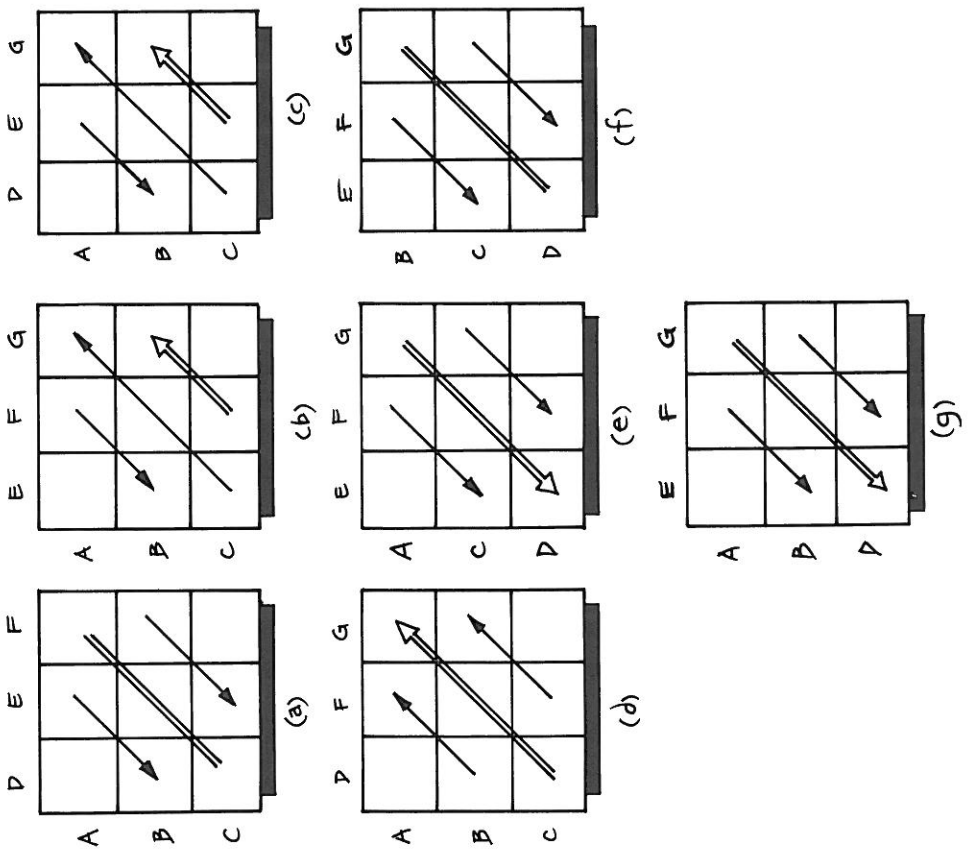


FIG. 7.14. The seven  $3 \times 3$  matrices contained within the upper half of Fig. 7.13 and the associated tests of double cancellation.

TABLE 7.1  
Numerical Estimates of A, B, C, D, E, F, and G and the Resulting  
Numerical Values of the Midpoints Between Them, Consistent  
With the Order Upon the Midpoints Shown in Fig. 7.13

|      | A=0 | B=5 | C=8 | D=16 | E=20 | F=22 | G=29 |
|------|-----|-----|-----|------|------|------|------|
| A=0  | .   | 2.5 | 4   | 8    | 10   | 11   | 14.5 |
| B=5  |     | .   | 6.5 | 10.5 | 12.5 | 13.5 | 17   |
| C=8  |     |     | .   | 12   | 14   | 15   | 18.5 |
| D=16 |     |     |     | .    | 18   | 19   | 22.5 |
| E=20 |     |     |     |      | .    | 21   | 24.5 |
| F=22 |     |     |     |      |      | .    | 25.5 |
| G=29 |     |     |     |      |      |      | .    |

What has been shown in this chapter is how the theory of conjoint measurement may be applied to Coombs' unfolding theory and an illustration of such application has been given. Should Coombs' theory prove true for at least certain stimulus domains, the theory of conjoint measurement could be used to establish standard sequences of stimuli that could then be used as a basis for psychological measurement.

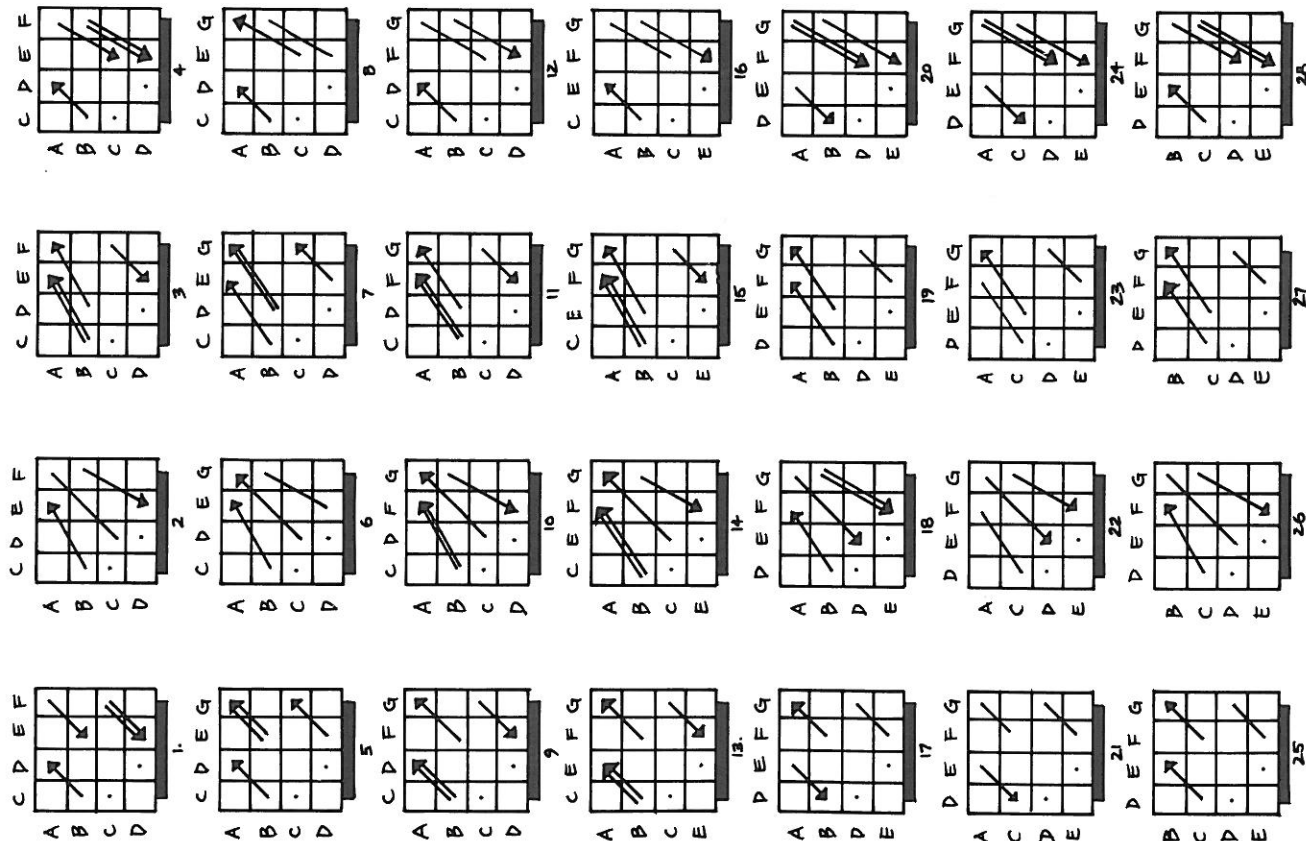


FIG. 7.15. Twenty-eight additional tests of double cancellation required of the order upon midpoints shown in Fig. 7.13. These additional tests are required in the special case where the values defining the six rows and columns are the same. As with other tests of double cancellation, the condition is falsified if and only if the middle arrow points in a direction opposed to both outside arrows.